The isoscalar monopole resonance of the alpha particle: a prism to nuclear Hamiltonians

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We present an ab-initio study of the isoscalar monopole excitations of ^{4}He using different realistic nuclear interactions, including modern effective field theory potentials. In particular we concentrate on the transition form factor $F_\mathcal{M}$ to the narrow 0^+ resonance close to threshold. $F_\mathcal{M}$ exhibits a strong potential model dependence, and can serve as a kind of prism to distinguish among different nuclear force models. Comparing to the measurements obtained from inelastic electron scattering off ^{4}He, one finds that the state-of-the-art theoretical transition form factors are at variance with experimental data, especially in the case of effective field theory potentials. We discuss some possible reasons for such discrepancy, which still remains a puzzle.

The isoscalar monopole strength of large nuclei has been extensively studied since the discovery of a giant monopole resonance in ^{144}Sm and ^{208}Pb [1]. The reason for the great interest in such excitations originates from their connection to the incompressibility modulus of infinite nuclear matter [2, 3]. The alpha particle is a light nucleus, that however has a binding energy per particle similar to that of large systems and a high central density. While it possesses no bound excited states, it exhibits a very pronounced narrow resonance (^{4}He*) with the same quantum numbers 0^+ as the ground state, i.e., an isoscalar monopole resonance. Today, the development of few-body theories has reached a point, where an ab-initio calculation of the four-body isoscalar monopole transition strength can be carried out with high precision. As will become evident in the following, the comparison of such four-body results with experimental data can serve as a stringent test for nuclear Hamiltonians, that are the sole ingredients of an ab-initio quantum mechanical approach.

The four-nucleon isoscalar monopole resonance is located at $E_R^{exp} = -8.20 \pm 0.05$ MeV, with a width of 270±50 keV [4], just above the first two-body break-up threshold $E^p_{thr} = -8.48$ MeV into a proton and a triton and below the next threshold $E^n_{thr} = -7.74$ MeV into a neutron and a ^{3}He. A summary about the experimental studies of the spectrum of ^{4}He can be found in Ref. [5]. Valuable information about the nature of the resonance is given by the transition form factor $F_\mathcal{M}(q)$ measured in electron scattering experiments (^{4}He$(e,e')^4$He*) at various momentum transfer q. Similarly to the case of the elastic form factor, the q dependence of $F_\mathcal{M}$ reflects the dynamics at various interaction ranges.

The progress in ab-initio few-body methods allows today to obtain accurate results for observables in light nuclear systems using realistic potential models (see review [6]). In recent years the debate regarding potential models has boosted, especially after the introduction of the effective field theory (EFT) strategy in nuclear physics [7]. At present, both phenomenological realistic and chiral EFT potentials are used in ab-initio calculations, but only for very few observables large differences are found, e.g., for the polarization observable A_y of p-^{3}He scattering [8]. In this letter, we point out that the calculated ^{4}He isoscalar monopole resonance transition form factor $F_\mathcal{M}(q)$ depends dramatically on the nuclear Hamiltonian. Thus, it can serve as a kind of prism to distinguish among nuclear force models.

Main Results. The isoscalar monopole strength $S_\mathcal{M}(q,\omega)$ is in general a function of q and the energy transfer ω. It is given by

$$S_\mathcal{M}(q,\omega) = \sum\!\!\!\!\!\!\int dn |\langle n|\mathcal{M}(q)|0\rangle|^2 \delta(\omega - E_n + E_0) \quad (1)$$
$$= -\frac{1}{\pi} Im \langle 0|\mathcal{M}^\dagger(q) \frac{1}{\omega - H + E_0 + i\epsilon} \mathcal{M}(q)|0\rangle \,,$$

where $|0\rangle, |n\rangle$ and E_0, E_n are eigenfunctions and eigenvalues of the nuclear Hamiltonian H, and

$$\mathcal{M}(q) = \frac{G_E^s(q)}{2} \sum_i^A j_0(qr_i) \,, \quad (2)$$

is the isoscalar monopole operator. Here $G_E^s(q) = G_E^p(q) + G_E^n(q)$ is the nucleon electric isoscalar form factor [9], $\boldsymbol{r}_i$ is the nucleon's position, and j_0 is the spherical Bessel function of 0^{th} order. The monopole strength can be written as a sum of a resonance term $S^{\rm res}_\mathcal{M}$ and a non-resonant background contribution $S^{\rm bg}_\mathcal{M}$,

$$S_\mathcal{M}(q,\omega) = S^{\rm res}_\mathcal{M}(q,\omega) + S^{\rm bg}_\mathcal{M}(q,\omega) \,. \quad (3)$$

For a narrow resonance one defines the resonance transition form factor

$$|F_\mathcal{M}(q)|^2 = \frac{1}{Z^2} \int d\omega S^{\rm res}_\mathcal{M}(q,\omega) \,. \quad (4)$$

In Fig. 1, we show results for $F_\mathcal{M}(q)$ with two different Hamiltonians including realistic three-nucleon forces (3NF) in comparison to experimental data from inelastic electron scattering [4, 11, 12]. As Hamiltonians we

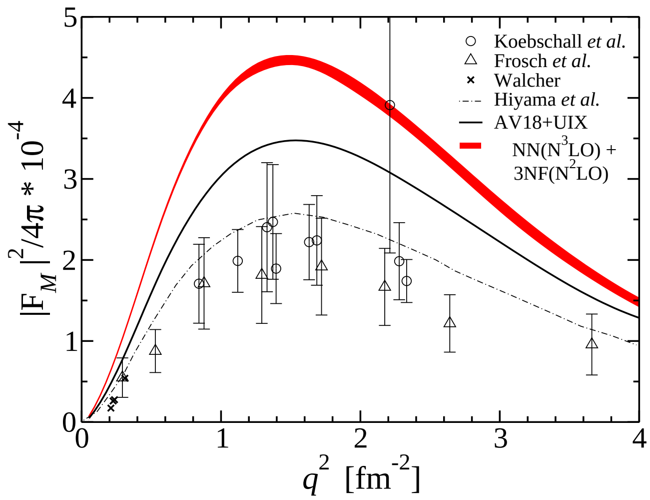

FIG. 1. (Color online) Theoretical transition form factor $|F_{\mathcal{M}}(q^2)|^2$ with $G_E^n = 0$ calculated with various force models: AV18+UIX (full line), N^{3}LO+N^{2}LO (red band); result from [10] (dot-dashed). Data from Frosch *et al.* [11], Walcher [4] and Köbschall *et al.* [12].

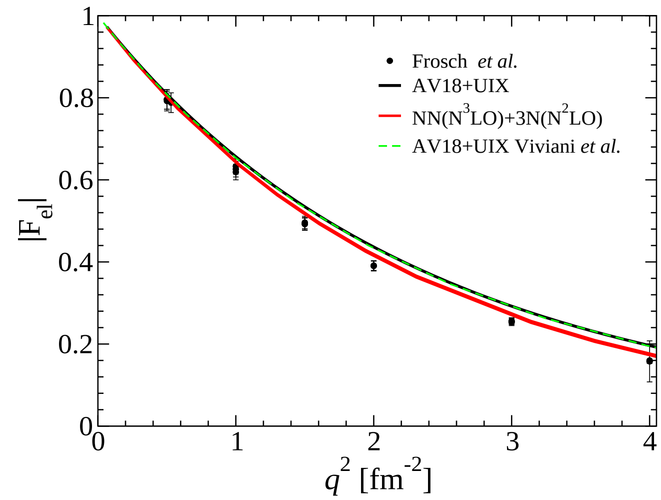

FIG. 2. (Color online) Elastic form factor $|F_{el}(q^2)|$ of ^{4}He calculated with various force models: AV18+UIX (full line); N^{3}LO+N^{2}LO (red band); result from [18] with AV18+UIX (dot-dashed). Data from Frosch *et al.* [19].

use (i) the AV18 [13] NN potential plus the UIX [14] 3NF, (ii) an EFT based potential, where we take the NN potential [15] at fourth order (N^{3}LO) in the chiral expansion augmented by a 3NF at order N^{2}LO [16]. The Coulomb potential is taken into account in all calculations. Both the EFT and the AV18 NN potentials reproduce the NN scattering phase shifts with high precision (χ^2/datum ≈ 1). In the EFT calculations, two different parameterizations of the 3NF have been used, leading to the red band in Fig. 1. The chiral low energy constants c_D and c_E have been determined either by setting c_D to a reasonable value and then fitting c_E to the three-nucleon binding energies [16] ($c_D = 1$ and $c_E = -0.029$) or by fitting to the ^{3}H binding energy and beta decay [17] ($c_D = -0.2$ and $c_E = -0.205$). We also display the result of a previous calculation by Hiyama *et al.* [10], with the AV8' potential, a reduced version of AV18, and a simplified central 3NF, fitted to the binding energy of ^{3}H. All three Hamiltonians reproduce the ^{4}He experimental binding energy within one percent. Surprisingly, the results for $F_{\mathcal{M}}(q)$ strongly depend on the Hamiltonian. Furthermore, the realistic Hamiltonians fail to reproduce the experimental data. In particular, this is true for the EFT forces that predict a transition form factor twice as large as the measured one.

In contrast, the realistic Hamiltonians lead to rather similar results for the elastic form factor $F_{el}(q)$ of ^{4}He, defined as

$$F_{\rm el}(q) = \frac{1}{Z} \langle 0 \,| \mathcal{M}(q) |\, 0 \rangle \ . \tag{5}$$

In Fig. 2, $F_{el}(q)$ is shown for the AV18+UIX model and for the chiral EFT potentials. The fact that the results do not differ significantly is not very surprising, since both Hamiltonians give a very similar result for the radius: 1.432(2) fm [20] for AV18+UIX and 1.464(2) fm for N^{3}LO plus the N^{2}LO of [17] which is not far from the experimental value of 1.463(6) fm (obtained from the charge radius of Ref. [21] as explained in [22]). Also shown in Fig. 2 is the result by Viviani *et al.* [18] with the AV18+UIX potential, which is indistinguishable from ours, proving the level of accuracy of contemporary four-body calculations.

TABLE I. Ground state energies in MeV for ^{3}H, ^{3}He and ^{4}He with N^{3}LO [15] and N^{2}LO (parameterizations from [16] or [17]). Comparison of present results (EIHH) with no core shell model (NCSM) and hyperspherical harmonics (HH) results.

3NF from [16]	EIHH	NCSM [16]	HH [25]	Nature
^{3}H	-8.474(1)	-8.473(5)	-8.474	-8.48
^{3}He	-7.734(1)		-7.733	-7.72
^{4}He	-28.357(7)	-28.34(2)	-28.37	-28.30
3NF from [17]	EIHH	NCSM [17]		Nature
^{3}H	-8.472(3)	-8.473(4)		-8.48
^{3}He	-7.727(4)	-7.727(4)		-7.72
^{4}He	-28.507(7)	-28.50(2)		-28.30

Calculational Method. Our calculations are carried out using the effective interaction hyperspherical harmonics method (EIHH) [23, 24]. In Table I, we present the EIHH binding energies of three- and four-body nuclei obtained from EFT potentials including 3NF. Comparing our results to other calculations we find agreement at the 10 keV level. Results for $S_{\mathcal{M}}(q,\omega)$ are often obtained by discretizing the continuum. In this case the Hamiltonian is represented on a finite basis of square integrable functions and is then diagonalized to obtain eigenvalues

e_ν and eigenfunctions $|\nu\rangle$. In this way one achieves an ill defined discretized representation of $S_{\mathcal{M}}(q,\omega)$. In our Lorentz integral transform (LIT) approach [26, 27] the continuum discretization is used to reach the continuum spectrum as follows. We calculate

$$\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma) = -\frac{1}{\pi} Im\langle 0|\mathcal{M}^\dagger(q)\frac{1}{\sigma - H + E_0 + i\Gamma}\mathcal{M}(q)|0\rangle\,, \tag{6}$$

where Γ is finite. It is easy to proove that $\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma)$ is connected to $S_{\mathcal{M}}(q,\omega)$ by an integral transform with a Lorentzian kernel $K(\omega,\sigma,\Gamma) = \frac{\Gamma}{\pi}\frac{1}{(\omega+E_0-\sigma)^2+\Gamma^2}$,

$$\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma) = \int d\omega\, K(\omega,\sigma,\Gamma)\, S_{\mathcal{M}}(q,\omega)\,. \tag{7}$$

Since Γ is finite $\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma)$ can be calculated with bound-state methods, namely it is legitimate to represent the Hamiltonian on a basis of square integrable functions. After having reached convergence in the energy region of interest, with N basis functions, the LIT of $S_{\mathcal{M}}(q,\omega)$ is given by

$$\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma) = \frac{\Gamma}{\pi}\sum_{\nu=1}^{N}\frac{|\langle\nu|\mathcal{M}(q)|0\rangle|^2}{(\sigma - e_\nu + E_0)^2 + \Gamma^2}\,. \tag{8}$$

In the next step an inversion of the transform (7) has to be performed to determine $S_{\mathcal{M}}(q,\omega)$. This same procedure has been followed in Ref. [28, 29] but for the complete nuclear charge operator. There, however, the resonance region was not considered.

The diagonalization of the Hamiltonian allows us to identify the resonance energy at $E_R = -7.40(20)$ MeV for AV18+UIX and $E_R = -7.50(30)$ MeV for N^{3}LO+N^{2}LO (extrapolated value). In fact we find that for $e_\nu = E_R$ the strength $S_R = |\langle\nu_R|\mathcal{M}(q)|0\rangle|^2$ is very pronounced. The strength S_R is composed of a transition to the resonance and a background contribution. As explained in the following, by a proper inversion of our convergent results for $\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma)$ at $\Gamma = 5$ MeV we have been able to separate both contributions, i.e. $F_{\mathcal{M}}(q)$ and $\mathcal{S}^{bg}_{\mathcal{M}}(q,\omega)$.

For the inversion of the LIT we have used a regularization procedure [30], which usually consists in expanding the response on a set of continuous functions and in fitting the calculated $\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma)$ on the corresponding linear combination of the transformed basis functions. In the presence of a narrow resonance an explicit resonance form must be added to the basis [31], e.g., a Lorentzian with width γ located at E_R, $\mathcal{S}^{res}_{\mathcal{M}}(q,\omega) = \frac{\gamma}{2\pi}\frac{f_R(q)}{(\omega-E_R+E_0)^2+(\gamma/2)^2}$. In the ideal case both the resonance strength and its width can be resolved in the inversion process. However, in our present case, one finds that the best fits are obtained with a vanishing width, reflecting the absence of states $|\nu\rangle$ in the vicinity of E_R. Therefore, we treat the resonance as a $\delta-$function, which we simulate numerically by setting γ to a very small value.

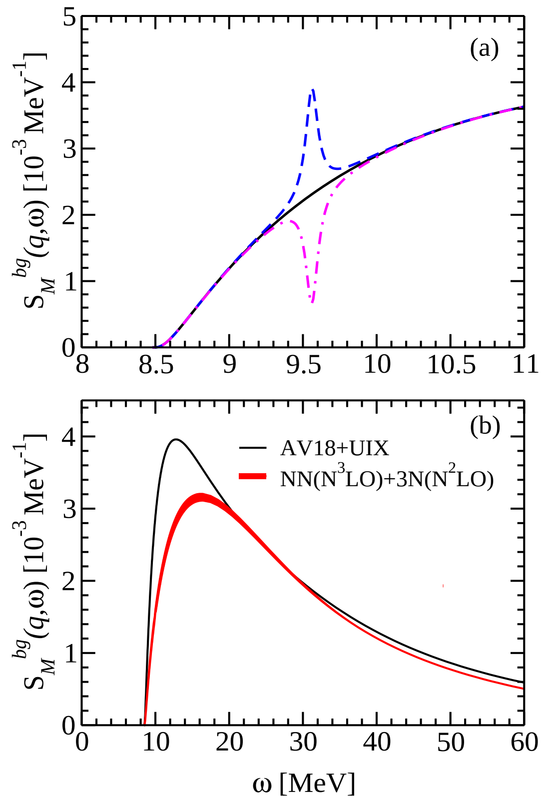

FIG. 3. (Color online) (a) $\mathcal{S}^{bg}_{\mathcal{M}}(q,\omega)$ at $q = 1.5$ fm^{-1} for AV18+UIX obtained with different values of f_R (see text): $f_R = |F_{\mathcal{M}}(q)|^2$ (full line), $f_R = 0.99|F_{\mathcal{M}}(q)|^2$ (dashed line); $f_R = 1.01|F_{\mathcal{M}}(q)|^2$ (dot-dashed line); (b) non-resonant background contribution $\mathcal{S}^{bg}_{\mathcal{M}}(q,\omega)$: AV18+UIX (full line); N^{2}LO+N^{3}LO (red band).

Since in our calculation the resonance turns out to be a $\delta-$function, the background LIT has the following form

$$\mathcal{L}^{bg}_{\mathcal{M}}(q,\sigma,\Gamma) = \mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma) - \frac{\Gamma}{\pi}\frac{f_R(q)}{(\sigma - E_R + E_0)^2 + \Gamma^2}\,, \tag{9}$$

where f_R remains to be determined in the inversion. Varying f_R one finds inversion results for $S^{bg}_{\mathcal{M}}$ that contain different resonance contributions. Results for various values of f_R are shown in Fig. 3a. The case where the resonance contribution vanishes corresponds to the correct value of the transition form factor $|F_{\mathcal{M}}(q)|^2 = f_R(q)/Z^2$ and the obtained strength is just $S^{\rm bg}_{\mathcal{M}}(q,\omega)$. We would like to emphasize that the results obtained are almost independent of the assumed resonance width γ, so long it remains sufficiently small. For the AV18+UIX potential the relative size of the background reduction, about 8%, is roughly q independent. For the N^{3}LO+N^{2}LO interactions the reduction varies between 13% for $q = 0.25\,\mathrm{fm}^{-1}$ and 22% for $q = 2\,\mathrm{fm}^{-1}$.

In Fig. 3b, the non-resonant monopole strength $S^{\rm bg}_{\mathcal{M}}$ is shown on a larger energy range, into the far continuum region. One sees quite a difference between the results with the EFT forces and the AV18+UIX potential. The

TABLE II. $|F_{\rm el}|$ and $S_R = |\langle \nu_R|\mathcal{M}(q)|0\rangle|^2$ for $q = 1.01$ fm^{-1} as a function of the grandangular momentum $K_{\max}$ with N^{3}LO+N^{2}LO [17].

$K_{\max}$	12	14	16	18
$\|F_{\rm el}\|$	0.6248	0.6244	0.6242	0.6241
$10^4 S_R/4\pi Z^2$	4.59	4.75	4.85	4.87

former lead to a lower low-energy peak and tail than the latter. These results show the power of the LIT approach, which enables one to calclate the strength in the far four-body continuum by reducing a scattering-state problem to a bound-state problem in a rigorous way.

Analysis of the Results. The main findings of this Letter are the dramatic sensitivity of $F_{\mathcal{M}}(q)$ to the nuclear Hamiltonian and the large deviations of realistic calculations from the available experimental data. These are unexpected results, so in the following we will try to analyze the possible sources for this puzzling situation. In Fig. 1 we presented all the available experimental data from Frosch *et al.* [11], Walcher [4] and Köbschall *et al.* [12]. $F_{\mathcal{M}}(q)$ has been obtained in inelastic electron scattering experiments measuring the longitudinal cross section at fixed values of q in the resonance region, subtracting the background contributions and integrating over the resonance structure. Even though one can contemplate the possibility of systematic experimental errors, the fact that the results correspond to three different sets of data, makes it less likely, so we will now discuss possible theoretical explanations.

(i) Our calculations are well converged in terms of the expansion of the wave functions. To elucidate this numerical aspect we show in Table II both $F_{\rm el}$ and S_R at $q = 1.01$ fm^{-1} for different grandangular momentum values $K_{\max}$, which is a measure of the wave function expansion in the EIHH method. Our results for S_R (and thus for $F_{\mathcal{M}}$) are converged at a few per cent level for all the implemented potentials, thus large numerical inaccuracies can be ruled out.

(ii) The isoscalar monopole operator in Eq. (2) is obtained from a multipole expansion of the usual one-body charge operator. The neglect of higher order operators (two- and three-body) is known as impulse approximation. The impulse approximation describes the elastic form factor very well up to a momentum transfer of $q = 2$ fm^{-1} (see Fig. 2). For the AV18+UIX potential it has been shown explicitly by Viviani *et al.* [18] that in ^{4}He two-body charge operators are negligible to $F_{\rm el}(q)$ below $q = 2$ fm^{-1}. This is not surprising since it is well known that two-body terms in the nuclear charge operator are of higher relativistic order. In fact, in EFT they appear only at fourth order [32]. Thus, they are unlikely to be the source of the large discrepancy between theory and experiment.

(iii) The EFT potentials we have used are a NN interaction at N^{3}LO and 3NFs at N^{2}LO. One may wonder whether N^{3}LO corrections to the 3NF can have a large effect. However, we notice that the overall effect of the 3NF on $F_{\mathcal{M}}(q)$ is rather mild (about 10%).

(iv) In our calculation the values of E_R given by both potential models (AV18+UIX, N^{3}LO+N^{2}LO) lie beyond the two thresholds and about 700 keV above the experimental result. On the other hand, the simplified force model used by Hiyama *et al.* [10] reproduces the experimental resonance position to within 100 keV, and at the same time predicts a smaller $F_{\mathcal{M}}(q)$, closer to the experimental data. One can envisage a correlation between the ability of a model to reproduce E_R and $F_{\mathcal{M}}$. In fact, if one considers that the transition form factor is the Fourier transform of the transition density from ^{4}He to ^{4}He*, one can imagine that small differences in E_R are reflected in the resonant wave functions and yield larger differences in the transition density. Similar conclusions have been drawn in Ref. [33] in the study of the differential cross section of p-^{3}H scattering. However, the resonant behaviour of the nuclear scattering amplitude is barely visible in the data. In contrast to the electromagnetic probe that amplifies the resonance signal considerably (see Fig. 1 of Ref. [12]).

Conclusions. We have calculated the isoscalar monopole ^{4}He $\longrightarrow ^4$He* transition form factor $F_{\mathcal{M}}(q)$ with realistic nuclear forces (N^{3}LO+N^{2}LO, AV18+UIX). Unexpectedly the results are strongly dependent on the Hamiltonian. Therefore this observable is ideal for testing nuclear Hamiltonians. As surprising as the large potential model dependence, is the fact that our $F_{\mathcal{M}}$ results are at variance with the experimental data, particularly large differences are found in the case of the chiral forces. It is very unlikely that corrections to the isoscalar monopole operator can lead to large effects. In order to clarify the situation it is highly desirable to have a further experimental confirmation of the existing data and in particular with increased precision. On the theory side further insight could be gained by an analysis of sum rules, densities, and different 3NFs. These issues will be the subject of future studies.

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[1] D.H. Youngblood *et al.*, Phys. Rev. Lett. **39**, 1188 (1977).
[2] O. Bohigas, A. M. Lane, and J. Martorell, Phys. Rep. **51**, 267 (1979).
[3] J.P. Blaizot, Phys. Rep. **64**, 171 (1980).
[4] Th. Walcher, Phys. Lett. B, **31**, 442 (1970).
[5] S. Fiarman and W.E. Meyerhof, Nucl. Phys. A, **206** 1 (1973); D.R. Tilley, H.R. Weller, G.M. Hale, Nucl. Phys. A, **541** 1-104 (1992).
[6] W. Leidemann and G. Orlandini, arXiv:1204.4617 ; Progr. Part. Nucl. Phys., in press.
[7] S. Weinberg, Phys. Lett. B **251**, 288 (1990); C. Ordez, L. Ray, U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994); E. Epelbaum, W. Gloeckle, U.-G. Meissner, Nucl. Phys. A **637**, 107 (1998).
[8] A. Kievsky, Few-Body Syst. **50**, 69 (2011).
[9] S. Galster *et al.*, Nucl. Phys. B **32**, 221 (1971).
[10] E. Hiyama, B.F. Gibson, and M. Kamimura, Phys. Rev. C **70**, 031001R-1 (2000).
[11] R.F. Frosch *et al.*, Phys. Lett. **19**, 155 (1965); Nucl. Phys. A, **110**, 657 (1968).
[12] G. Köbschall *et al.*, Nucl. Phys. A, **405**, 648 (1983).
[13] R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[14] B.S. Pudliner *et al.*, Phys. Rev. Lett. **74**, 4396 (1995).
[15] D.R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
[16] P. Navrátil, Few Body Syst., **41**, 117 (2007).
[17] D. Gazit, S. Quaglioni, and P. Navratil, Phys. Rev. Lett. **103**, 102502 (2009).
[18] M. Viviani *et al.*, Phys. Rev. Lett. **99**, 112002 (2007).
[19] R.F. Frosch *et al.*, Phys. Rev. **160**, 874 (1967).
[20] D. Gazit *et al.*, Phys. Rev. Lett. **96** 112301 (2006).
[21] I. Sick, Phys. Rev. C **77**, 041302R (2008).
[22] The charge radius $r_{\rm c}$ is converted to a point-proton radius $r_{\rm pp}$ by $r_{\rm pp}^2 = r_{\rm c}^2 - R_p^2 - (N/Z)\cdot R_n^2 - 0.033$ fm^2 where $R_p = 0.877(7)$ is the proton finite size, R_n^2=-0.1161(22) is the neutron finite size correction and the last term is a relativistic correction.
[23] N. Barnea, W. Leidemann, and G. Orlandini, Phys. Rev. C **61**, 054001 (2000); Nucl. Phys. A **693**, 565 (2001).
[24] N. Barnea and A. Novoselsky, Phys. Rev. A **57**, 48 (1998); Ann. Phys. (N.Y.) **256**, 192 (1997).
[25] A. Kievsky *et al.*, J. Phys. G **35**, 063101 (2008).
[26] V. D. Efros, W. Leidemann, and G. Orlandini, Phys. Lett. B **338**, 130 (1994).
[27] V. D. Efros *et al.*, J. Phys. G, **34**, R459 (2007).
[28] S. Bacca *et al.*, Phys. Rev. C 80, 064001 (2009).
[29] S. Bacca *et al.*, Phys. Rev. C **76**, 014003 (2007).
[30] A.N. Tikonov and V.Y. Arsenin, *Solutions of Ill–Posed Problems*, Washington, DC: V H Winston and Sons, 1977.
[31] W. Leidemann, Few-Body Syst. **42**, 139 (2008).
[32] T.-S. Park *et al.*, Phys. Rev. C **67**, 055206 (2003).
[33] R. Lazauskas, Phys. Rev. C **79**, 054007 (2009).